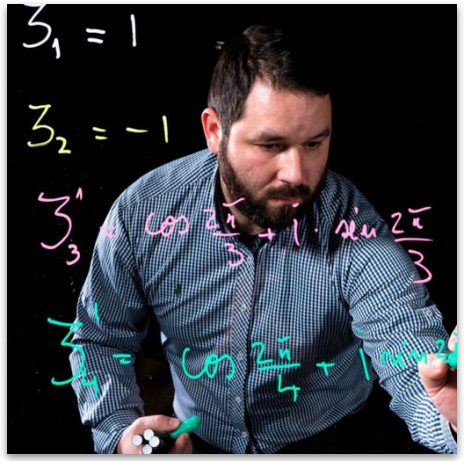


Mathematical education & problem solving: From zero to infinity, and real world applications

Matematica – motorul stiintei contemporane: viziune, metode, inovatie

Ovidiu Bagdasar
o.bagdasar@derby.ac.uk

Erasmus+

UNIVERSITY OF DERBY

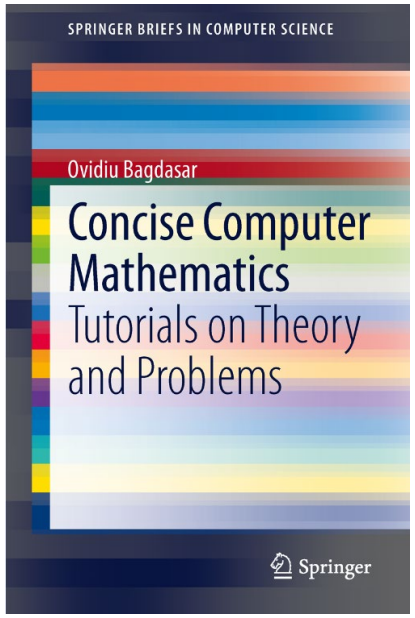
THE WORLD UNIVERSITY RANKINGS

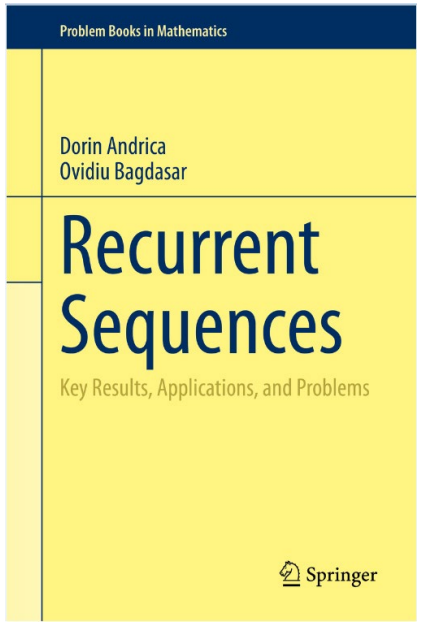
TEF Gold

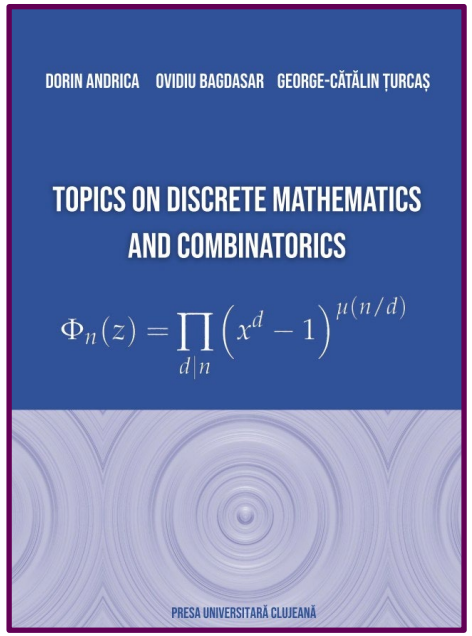
Vretta

Ovidiu

- Associate Professor in Mathematics
- PhD in Mathematics: Applied (2011), Pure (2015)
- PL MSc Big Data Analytics
- Erasmus/Turing coordinator
  - Romania: UBB, UAB, UVT, UAIC, UPB







Free download

University of Derby (UoD)

- **Derby:** Birthplace of the Industrial Revolution
- **Industry:** Planes, trains & automobiles (Rolls-Royce, Bombardier, Toyota)

The University

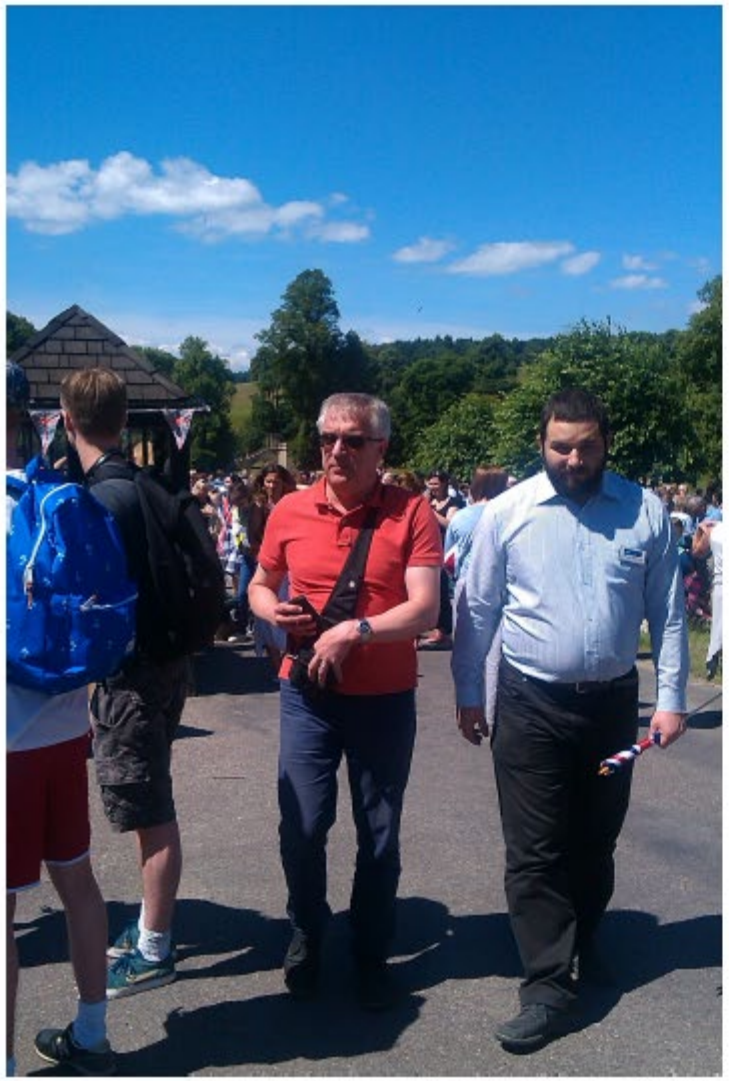
- Top 10% in the Knowledge Exchange Framework (KEF, 2021)
  'Working with Business' and *'Local Growth and Regeneration'*
- **Awarded Gold** in Teaching Excellence Framework (TEF)
- Key strengths: Business, Engineering, Hospitality, Computing

What makes Derby special

- Student centred (BBC)
- Employer engagement
- Research inspired teaching

Dedication to my mentors

Nicu Popovici, Convex Analysis



Preda Mihailescu (left)
Doru Andrica (right)
Catalan's conjecture

What is mathematics?

Roger Bacon (1214-1294): *“Mathematics is the key and the door to the Sciences*"

Galileo Galilei (1564-1642): "*The Universe is written in the language of mathematics*“

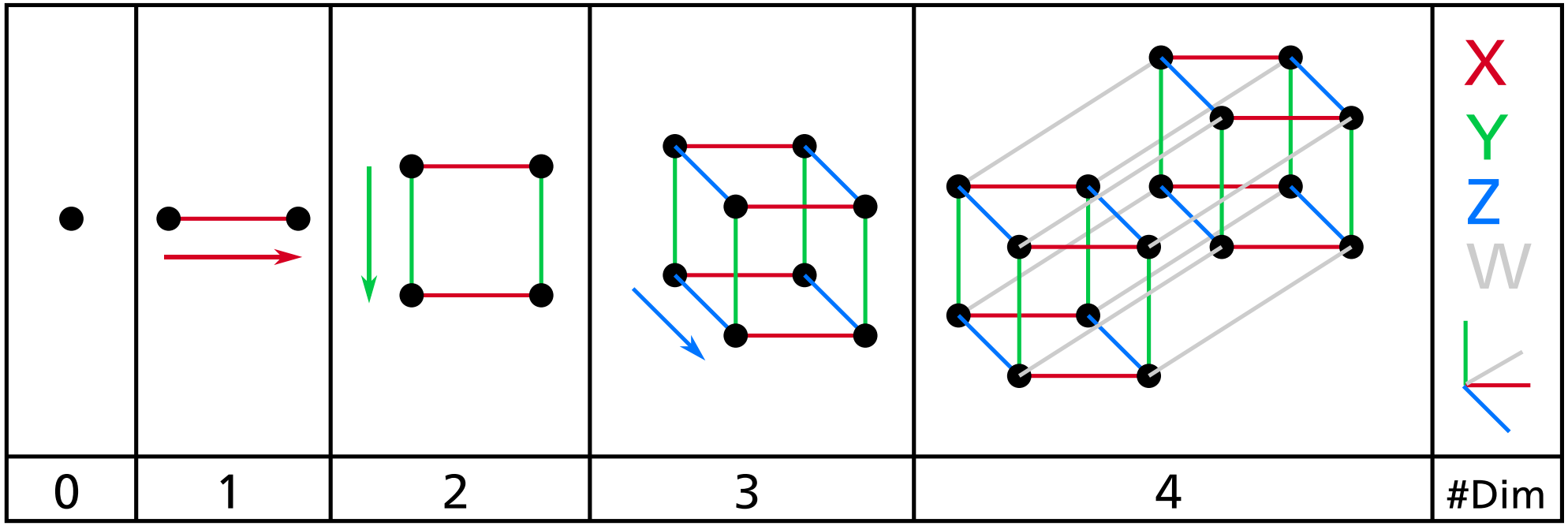
Martin Luther (1483 - 1546):
"*Medicine makes people ill,*
mathematics makes them sad*,*
and theology makes them sinful.“

Einstein:
"*Do not worry about your problems with mathematics... I assure you that mine are far greater*."

G. H. Hardy: "*There is no place in the world for ugly mathematics.*"

How big is Mathematics?
Check out MSC2020

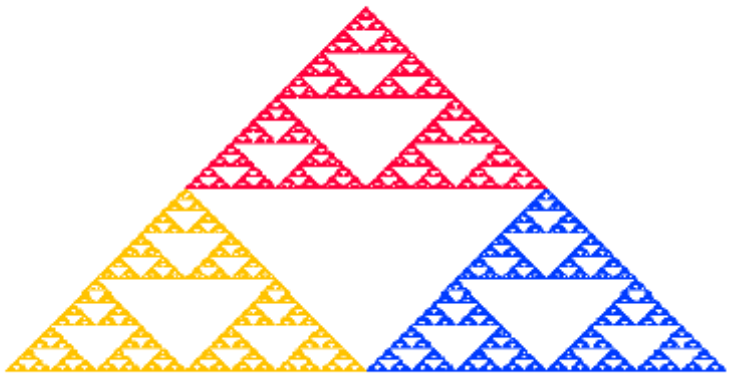
Maths concepts: Dimension



Fractals

- Self-similar patterns
- Mandelbrot (1975): "fractus" (latin: broken, fractured)
- "beautiful, damn hard, increasingly useful. That's fractals."

Sierpinski triangle



A_0 aria of the original triungle
P_0 perimeter of the original triangle

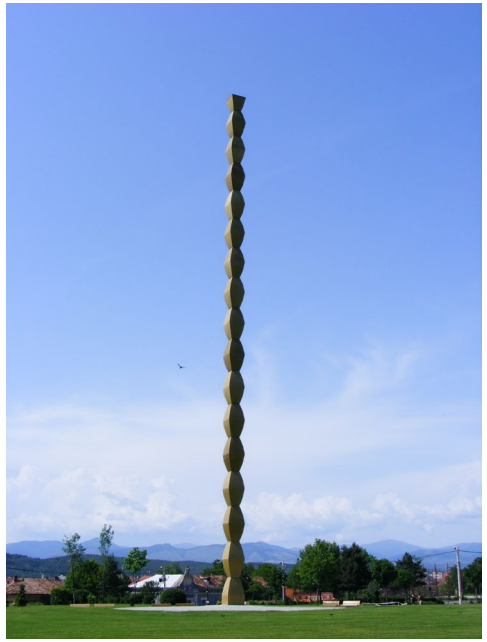
$$A_1 = \frac{3}{4}A_0, \ldots, A_n = \left(\frac{3}{4}\right)^n A_0.$$

$$P_1 = \frac{3}{2}P_0, \ldots, P_n = \left(\frac{3}{2}\right)^n P_0.$$

Conclusion

- Zero area
- Infinite perimeter
- Question: What is the dimension ?
- Hausdorff: $\frac{\log(3)}{\log(2)} \sim 1.585$

From zero to Infinity



Beginning:

- Antiquity: Actual/potential infinity, Arhimedes, Euclid (primes)
- John Wallis: ∞ (1655)

Infinity in nature and poetry/theology

- Number of atoms in the Universe: $10^{78} - 10^{82}$
- Number of possible chess games: 10^{40}

Sets: Naturals ($\mathbb{N}$), Integers ($\mathbb{Z}$), Rationals ($\mathbb{Q}$), Reals ($\mathbb{R}$)

Cardinal: Number of elements. Finite/infinite sets

Paradoxes of infinity: The "battle" of cardinals

$$\{0,1,2,3,\dots\} \quad \textit{vs.} \quad \{1,2,3,4,\dots\} \qquad "\infty = \infty - 1" \qquad f(n) = n+1$$

$$\{1,2,3,\dots\} \quad \textit{vs.} \quad \{2,4,6,\dots\} \qquad "\infty = \infty/2" \qquad f(n) = 2n$$

$$\mathbb{N} \quad \textit{vs.} \quad \mathbb{Q} \qquad "\infty = \infty \times \infty"$$

Rational numbers are countable (G. Cantor, 1874)

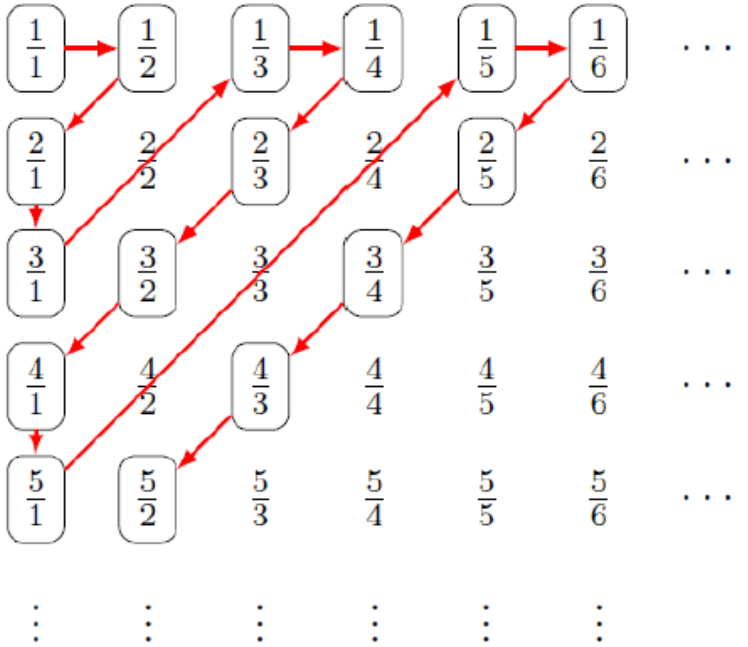
Sets:

- Natural numbers $\mathbb{N}$: $\{0, 1, \ldots, \}$
- Rational numbers $\mathbb{Q}$: fractions

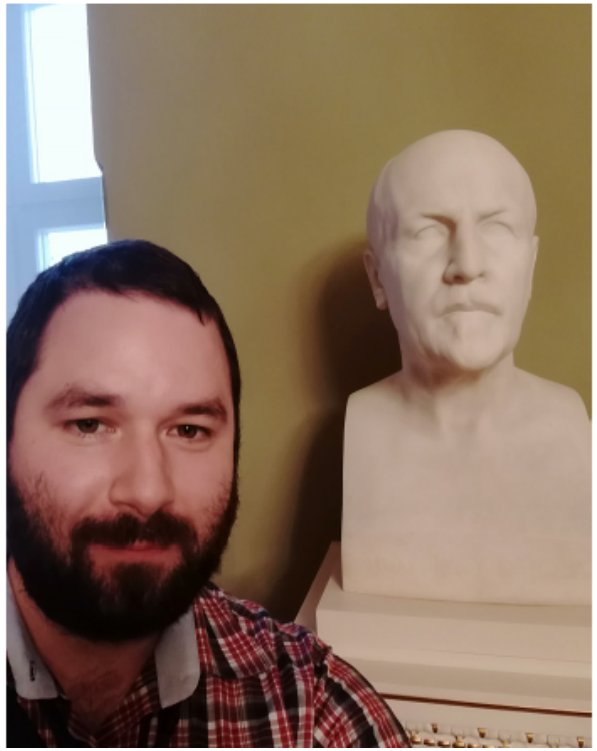
Conclusion: Rational numbers are as many as the natural numbers.

Argument: We can establish a bijective correspondence (1-to-1).

Countability of rationals



Selfie with Cantor



Leopold Kronecker: *"I don't know what predominates in Cantor's theory — philosophy or theology, but I am sure that there is no mathematics there."*

Real numbers are not countable (G. Cantor, 1874)

Assume that you can count the real numbers.

$$a_1 = x_{10}.x_{11}x_{12}x_{13}\ldots x_{1n}\ldots$$
$$a_2 = x_{20}.x_{21}x_{22}x_{23}\ldots x_{2n}\ldots$$
$$a_3 = x_{30}.x_{31}x_{32}x_{33}\ldots x_{3n}\ldots$$
$$\ldots \quad \ldots$$
$$a_n = x_{n0}.x_{n1}x_{n2}x_{n3}\ldots x_{nn}\ldots$$
$$\ldots \quad \ldots$$

I am far from claiming my discoveries are due to personal merit, because I am only an instrument of a higher power that will continue to work long after me, just as it revealed itself thousands of years ago to Euclid and Archimedes.

There is $b = b_0.b_1\ldots b_n\ldots$ cu $b_n \neq x_{nn}$, $n \geq 1$, so, b is not in the list.

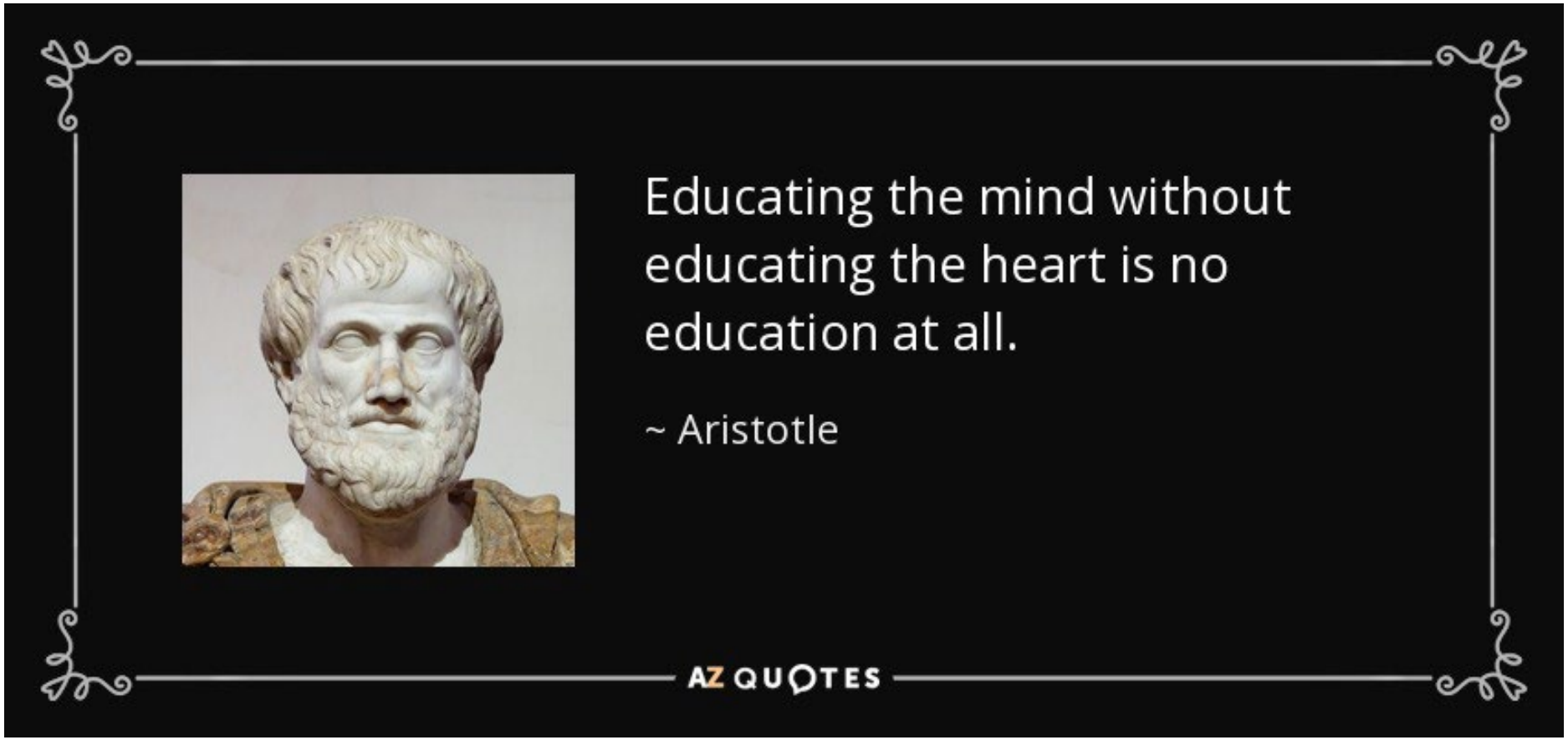
Conclusion:

- $\mathbb{N} \sim \mathbb{Z} \sim \mathbb{Q} \sim$ algebraic numbers $\quad \aleph_0$
- $\mathbb{N} < \mathbb{R}$ $\quad \aleph_1$
- $0, 1, 2, 3, \ldots, n, \ldots; \aleph_0, \aleph_1, \aleph_2, \ldots$

David Hilbert: "No one shall expel us from the Paradise that Cantor has created."

Education and Mathematics

Forbes: “Education's purpose is to replace an empty mind with an **open one**.“



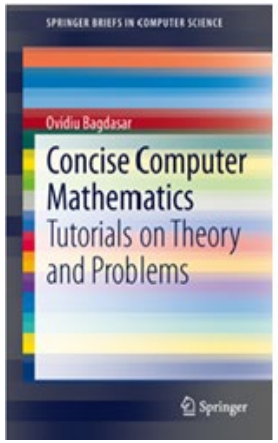
In STEM, the true Art is touching the HEART.

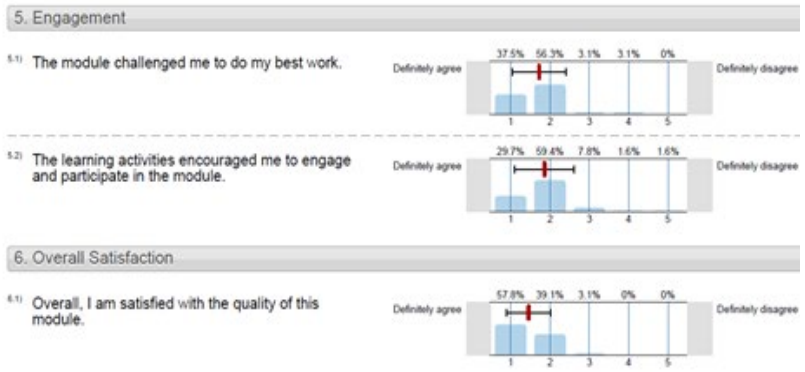
Maths Education approaches

Module leadership (2-3 modules / term)

Innovative practice: Technology

- Textbook + Recordings + Maths Jokes
- E-assessment and numeracy skills training @UoD
- Interactive notes, live polls, peer-learning…
- Blog article link posted in my lecture chat.





- NSS: 100% in 2019 & 2020, 94% in 2021
- TEF: Subject lead for Mathematics (2019)
- Nominated for NTF (2018, 2021)
- Fellow of the IMA (2019-)
- Senior Fellow of HEA (2020-)
  - Internationalization in HE
  - Technology in Mathematical Education

Vretta Maplesoft DigitalEd

Talks: TALMO 21 • Advance HE L&T 2021 • ALTC 21 • EAMS 2021 • 40th OCMA

Erasmus+

TEF Gold

Teaching abroad: 15+ Erasmus visits (Romania, Turkey, China – KA107)

Problem solving

M. Yates: Job $\Leftrightarrow$ Problem solving skills

Question: Are Maths students aware of this skill ?

The steps of problem solving
("How to Solve It", Polya, 1957):

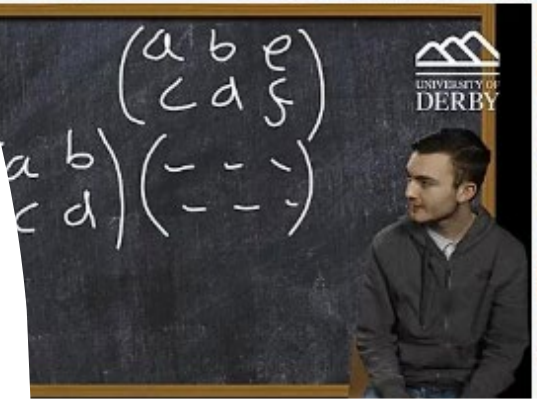
1. Understand the problem.
2. Make a plan.
3. Carry out the plan.
4. Look back on your work.
   How could it be better?

Bill Gates: "Now more than ever, an education that emphasizes general problem solving skills will be important."

Getting Maths Support right

- **Quality maths education improves**
  - Student attainment and performance
  - HE Evaluations: NSS, REF, KEF
  - Employability

- **Ambition to provide Maths Support**
  - Maths support for all UoD students and staff
  - Teaching support (mentors & technology)
  - Outreach activities in local schools
  - Support the wider society



@BagdasarO

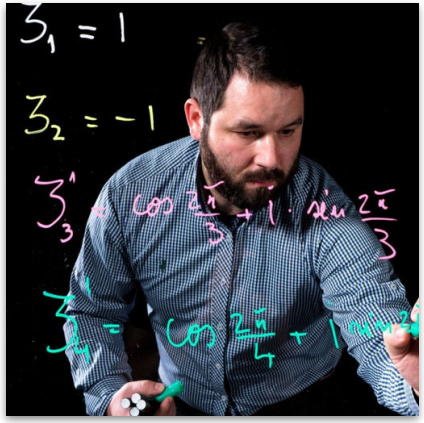
Boundary Leapers keynotes on Maths and English @LandauSCITT. Such a joy and privilege to inspire young children, and to work with the excellent Maths Mentors from @DerbyMaths and the Widening Access team @DerbyUni. @CharlotteWA_UoD @DerbyOppArea

5:51 PM · Feb 26, 2020 · Twitter for Android

View Tweet activity

5 Retweets 17 Likes

Using digital assessment in mathematics to improve university-wide student retention



Ovidiu Bagdasar
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by Bett ahead

a Hyve event

Matt Wingfield
matt.wingfield@e-assessment.com

UNIVERSITY OF DERBY

Sandbox Theatre, ExCel, London
Friday 31st March, 2023

eAA the e-Assessment Association

Current issues

1) Apr 2018 – UoD Workshop: *“Current issues in Mathematical Education“*
2) Dec 2018 – UoD Workshop: *"Effective Maths Education: Improving accessibility, reducing anxiety”*

Topics: Maths Anxiety, Teacher Development, Assessment, Roles of technology.
Participants: Researchers, Teachers, Vretta, ITS, IMA, Spaghetti Maths
Discussions on how to impact REF, TEF, KEF, NSS,…

Image description: 2nd UoD Workshop, Dec 2018

Focus at that time

- Maths Anxiety
- Students & Maths A-Levels (~15%)

The really big question:
How do we support the other 85%?

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The Numeracy Gap Challenge

UNIVERSITY OF DERBY

Current Level

- Attitudes
- Myths
- Calculators
- **COVID-19**

Required Level

£20 bn (UK, 2019)

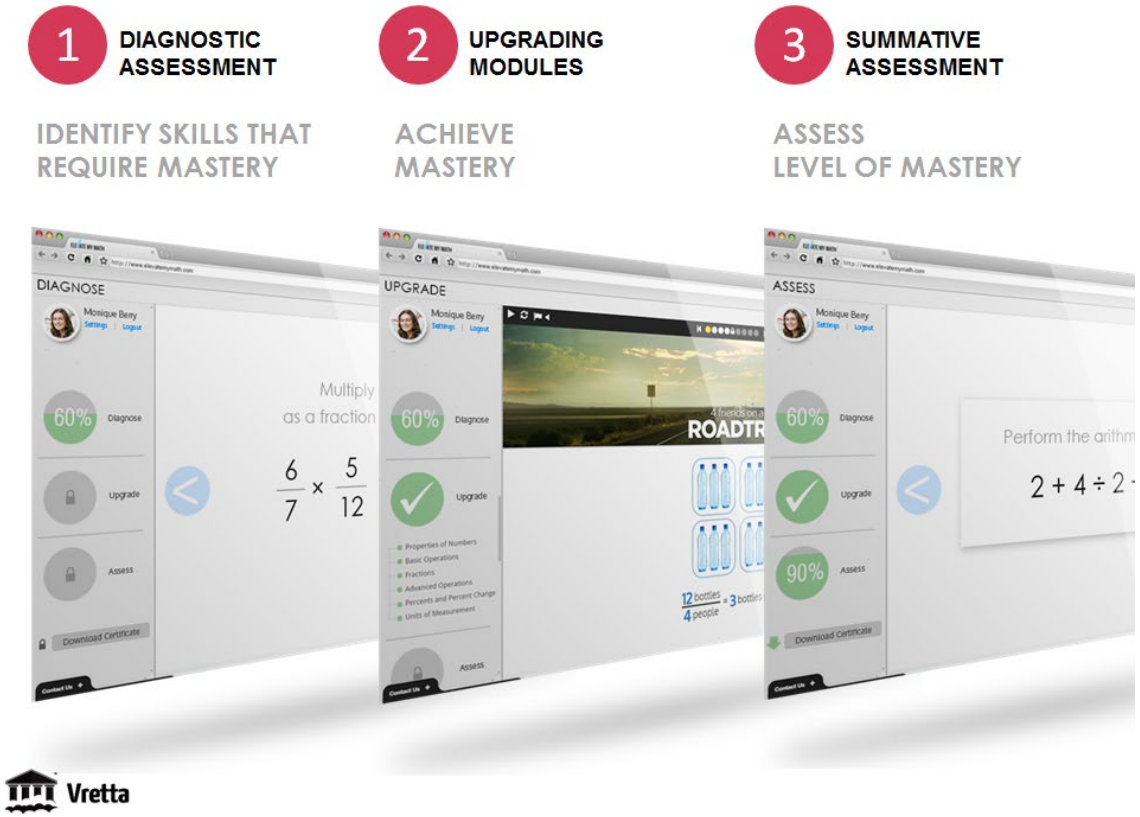
$90 bn (USA, 2021)

49%

OF BRITISH ADULTS ARE BELOW THE DESIRED LEVEL OF NUMERACY*

The Elevate My Maths Solution

Flexible, Engaging Assessment for Learning



ELEVATE MY MATHS

- Basic maths for College students
- Fills gaps in prior knowledge
- Assessment for learning

Products: EMM (Canada, UK), Mathematic (Luxembourg), IntroMath, etc.

EMM 1.0 Implementation at Derby (2018-19)

Advanced Numeracy Skills (60 questions)

College of Engineering & Technology

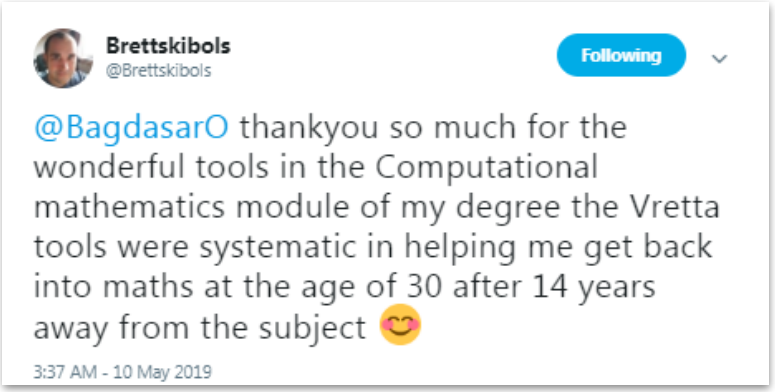
- Mathematics Year 1 (30)
- Computing Year 1 (130)
- Foundation Year 0 (110)
- Computing Med College (30)

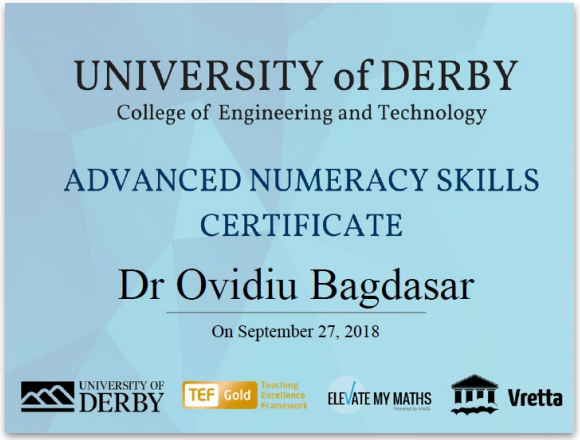
ELEVATE MY MATHS

Module KPI's:

- **Engagement:** 102 min (Pre-Test), 230 min (Remediation), 46 min (Post Test)
- **Improvement:** 14% Maths, 17% Year 0, Comp Maths (20% UoD, 39% Athens)
- **Pass rate (4CC503):** *Up to 90%, from 84-85% in previous years.*

- **Student Feedback:** *"Found it very useful as I hadn't studied Math for over 25yrs. I had a lot of remedy videos to get through, and spent many hours going through additional learning resources."*





Advance HE STEM 2019

UoD – Vretta partnership

UoD: Focused on Teaching Excellence, Research and Employability

Vretta: Canadian EdTech company specialized in Maths training.
"Our vision is a world where everyone enjoys maths" (Vretta Inc.)

The University of Derby becomes Vretta's first Academic Hub in the UK to Support Student Success in Mathematics

2018

First steps,
Pilot Project
300 Students
5 Courses

2019

Modularization
10 Courses
Feedback surveys
UoD Partnership 2019-2022

2020

Integration in LMS
100+ courses
Numeracy Badges
800 students

2021

ESF Training
SME Upskilling
Case Studies
Pedagogic Research

2022

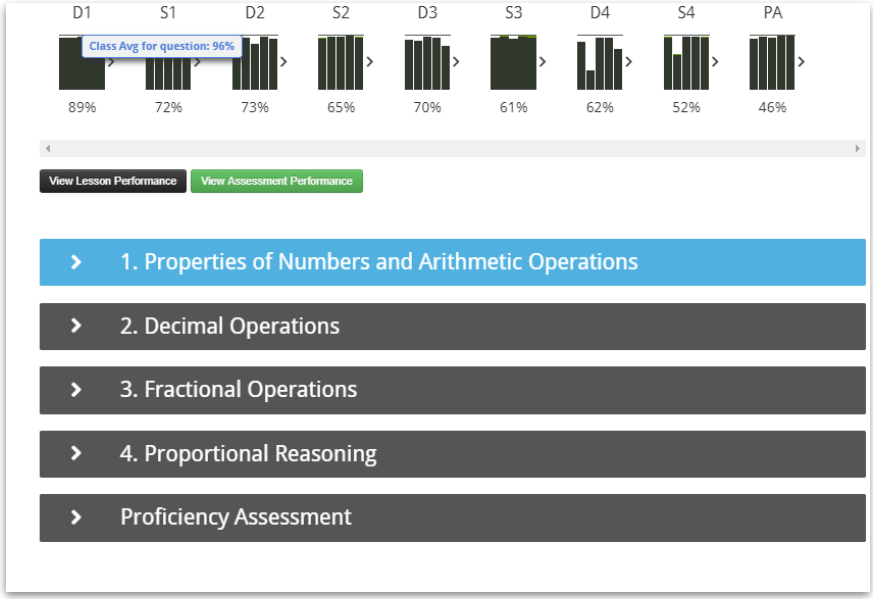
UoD Partnership 2022-2026
Local collaboration
International Expansion
Rolls Royce Nuclear Academy

UNIVERSITY OF DERBY Vretta

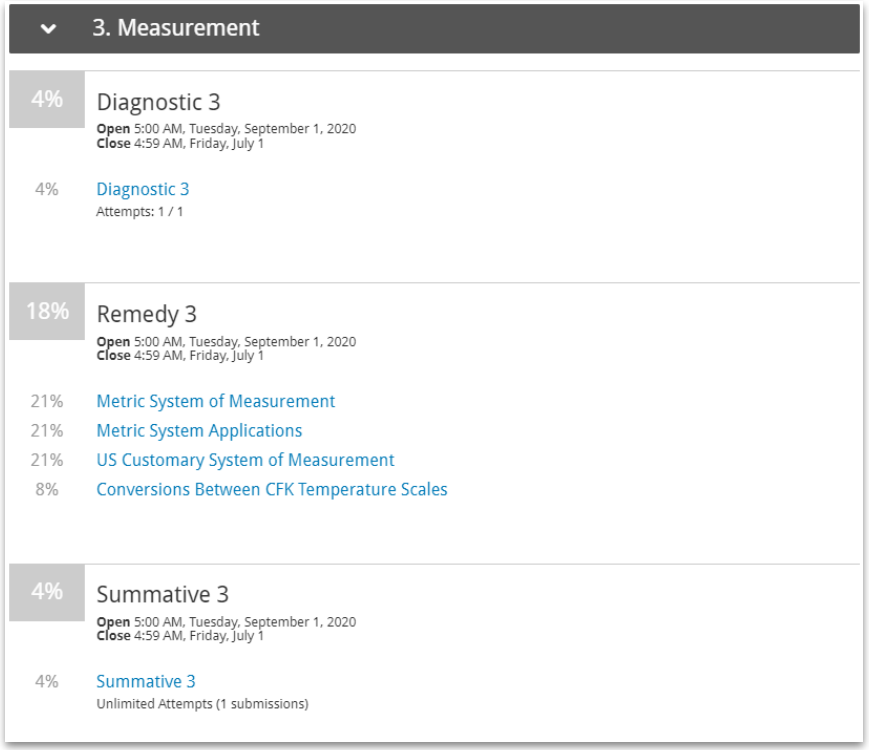
Elevate My Maths 2.0

UNIVERSITY OF DERBY

Modular Design



Formative Looping



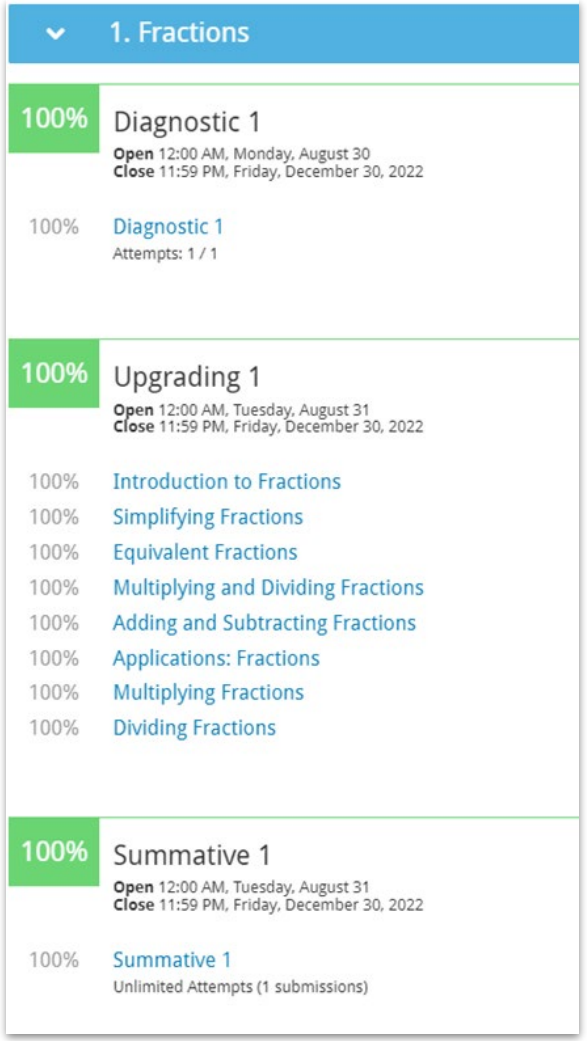
EDI Formative Examples

Innovation through collaboration

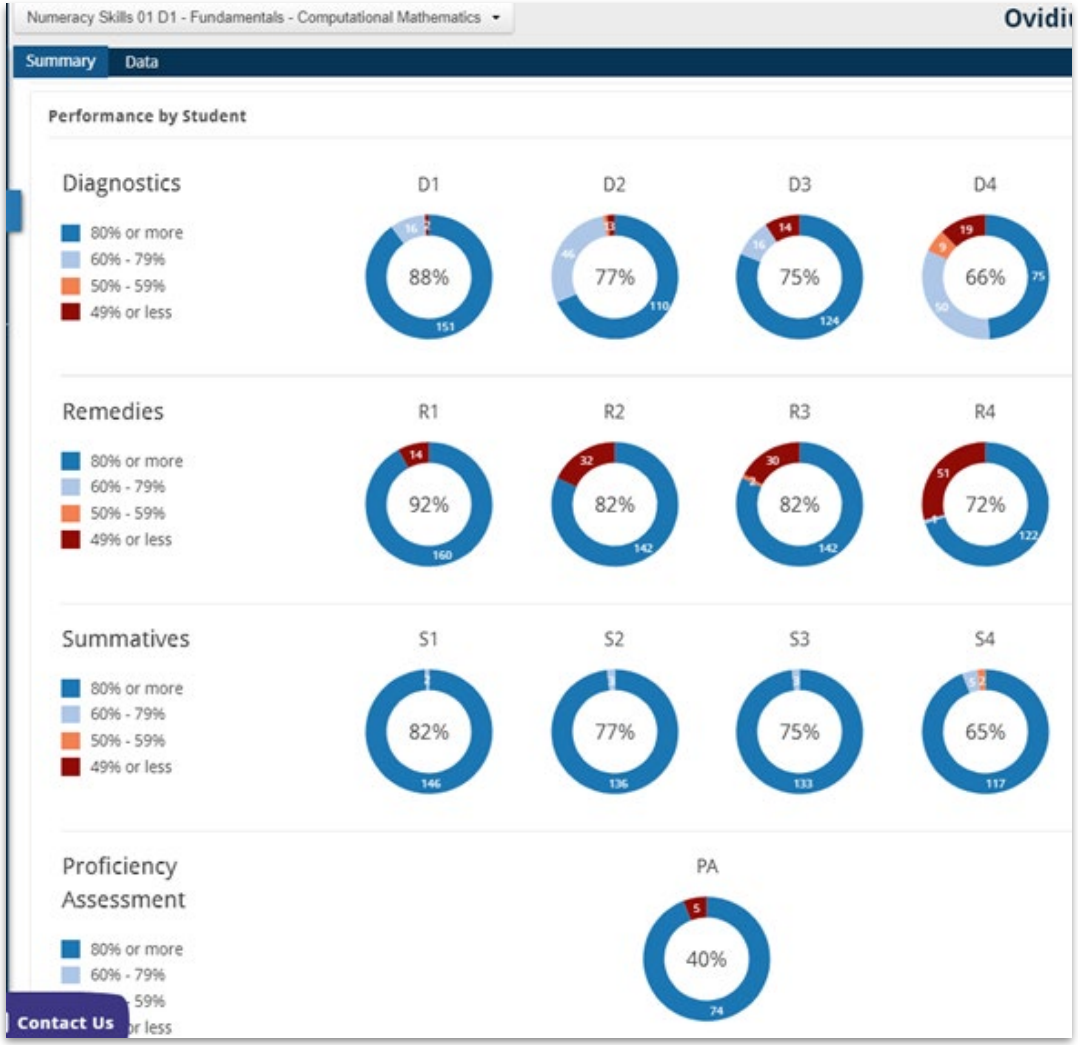
- Modular design (4-7 Topics)
- Topics with diagnostics, remedy lessons, summative assessments
- Assessment for learning: Repeatable summative tests (interactive, engaging)
- Proficiency Assessments & Digital Badges
- Interactive dashboards

Supporting tutors

Topic Module Complete



Dashboard View



D1 (Comp Maths 2020-21)

- (R1) 174: Engagement
- (S4) 128: Achievement
- (PA) 74: Completion

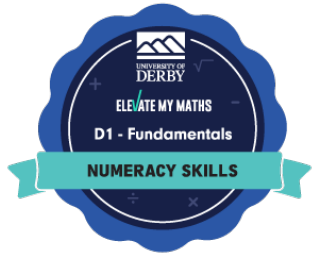
- KPI's at a glance, clickable dashboard features to get more granular detail as required.

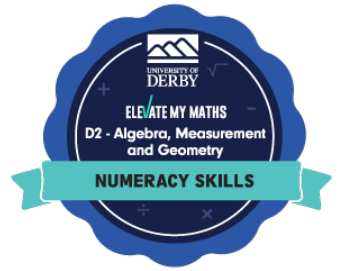
Impact in the classroom: EMM Case study

- 4CC503 Computational Mathematics performance
- KPIs: First pass rate (40%+), “good grades” (60%+).

4CC503 Computational Mathematics		Module reports		EMM Impact	
		Stud No	1st Pass (>=40%)	AB (>=60%)	1st Pass (>=40%)	AB (>=60%)
2015-16		168	140	99	83%	59%
2016-17		148	125	90	84%	61%
2017-18		133	115	77	86%	58%
2018-19	All	122	108	71	89%	58%
	EMM	107	99	69	93%	64%
2019-20	All	180	160	131	89%	73%
	D1	148	139	119	94%	80%
	D1&D2	123	121	107	98%	87%
2020-21	All	159	141	112	89%	70%
	D1	137	125	101	91%	74%
	D1&D2	113	108	94	96%	83%

Badge





Social Media Share

Numeracy training can help students build immunity against course failure.

Impact outside the classroom

Delivered the @DerbyUni course "Getting to Grips with Data: Numeracy and Problem Solving Skills" to @SadaccaLimited in Sheffield, and got my own badge. openbadgefactory.com/v1/assertion/5... ✅
The courses are free, and sponsored by the ESF project. #OpenHouseBadgeFactory @DerbyOppArea

8:00 AM · Apr 11, 2022 · Twitter Web App

European Union
European Social Fund

In partnership with Department for Work & Pensions

"The course was really powerful and practical. It helped me to refresh my numeracy skills whilst teaching me how to apply mathematical concepts to solve workplace problems. I believe everyone in business would benefit from learning these skills."

Olivier Tsemo
Chief Executive Officer, SADACCA

SADACCA

B & H ASSOCIATION DERBY

ORGANIZAȚIA ROMÂNILOR DIN EAST MIDLANDS

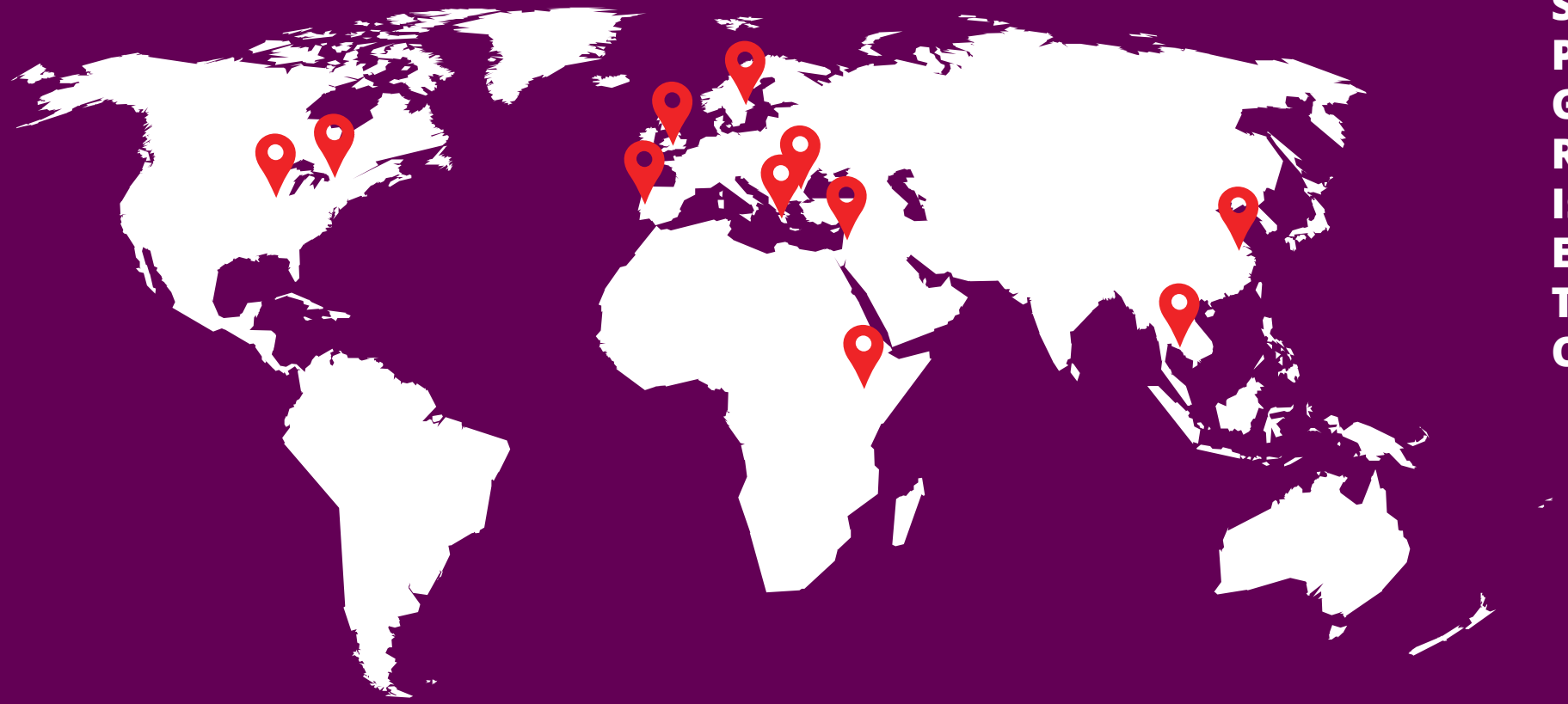
Rolls-Royce Nuclear Academy

GETTING TO GRIPS WITH DATA
NUMERACY AND PROBLEM SOLVING SKILLS
European Union
European Social Fund
UNIVERSITY OF DERBY
ELEVATE MY MATHS

Recognition of Innovation: 2022

e-Assessment Awards 2022
BEST FORMATIVE ASSESSMENT PROJECT
FINALIST
Vretta
UNIVERSITY OF DERBY

EMM Footprint

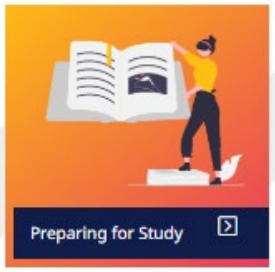


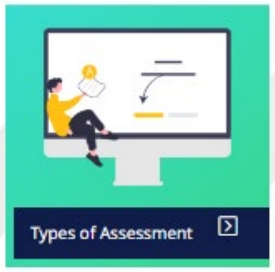
Canada
USA
UK
Sweden
Portugal
Greece
Romania
Israel
Ethiopia
Thailand
China

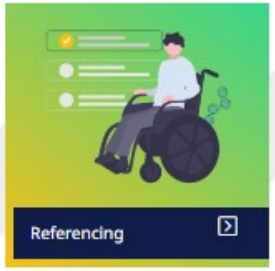
Develop@Derby

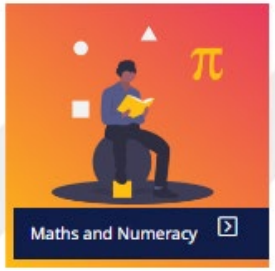
UNIVERSITY OF DERBY

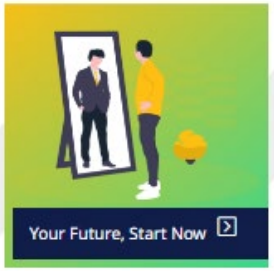
Develop@Derby is your one-stop shop for building personal skills that help with your university studies and beyond.











Maths & Numeracy Support

- 70+ modules, 3000+ students, 700+ badges
- Math Skills MOOC for UoD Applicants
- SME Upskilling for the workplace (ESF)

Pedagogic Research

- Erasmus+ KA2 grant (March 2019)
- REF 2021 Impact Case Study - UoA4 Psychology
- ALT-C, EAMS, BMC, eAA, AdvanceHE, TALMO, BETT

eAA the e-Assessment Association

ahead by Bett / A Hyve Event HIGHER EDUCATION

Research work

Fundamental research

- Nonlinear convex analysis and optimisation
- Recurrence sequences and dynamic geometry
- Computational number theory & combinatorics


PLOS ONE

OPEN ACCESS PEER-REVIEWED

RESEARCH ARTICLE

Disrupting resilient criminal networks through data analysis: The case of Sicilian Mafia

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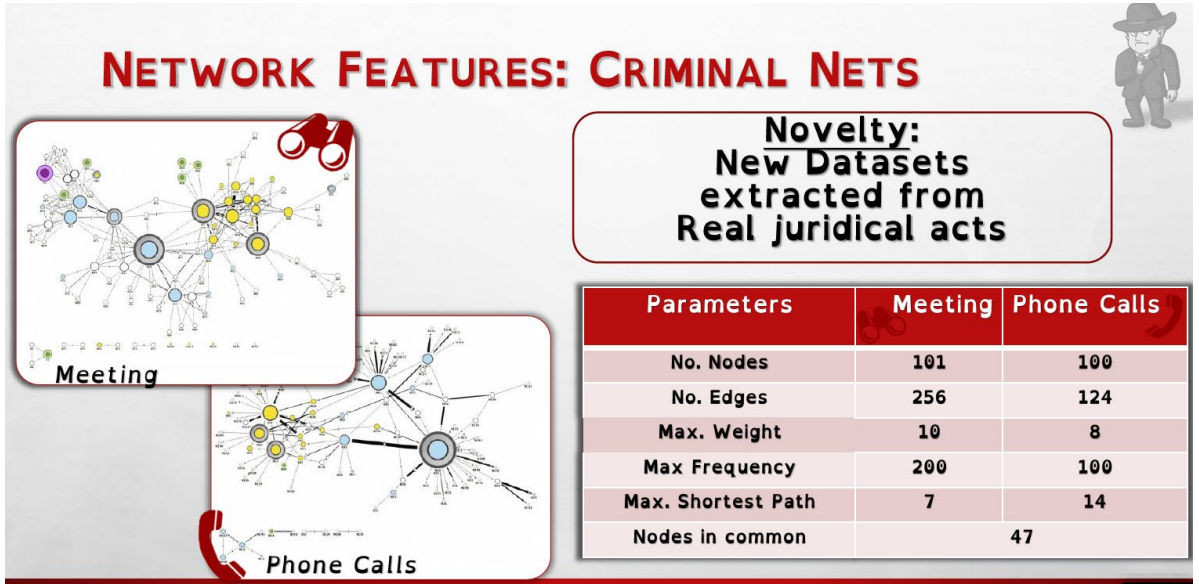
Article | Authors | Metrics | Comments | Media Coverage

Applied research

- Mathematical modelling (traffic, ceramics)
- Neural networks, maths in data science
- The analysis of criminal networks



Parameters	Meeting	Phone Calls
No. Nodes	101	100
No. Edges	256	124
Max. Weight	10	8
Max Frequency	200	100
Max. Shortest Path	7	14
Nodes in common	47	

Pedagogical research

- Mathematical anxiety
- <u>Closing the numeracy gap</u>
- Teaching mathematics ONLINE in STEM

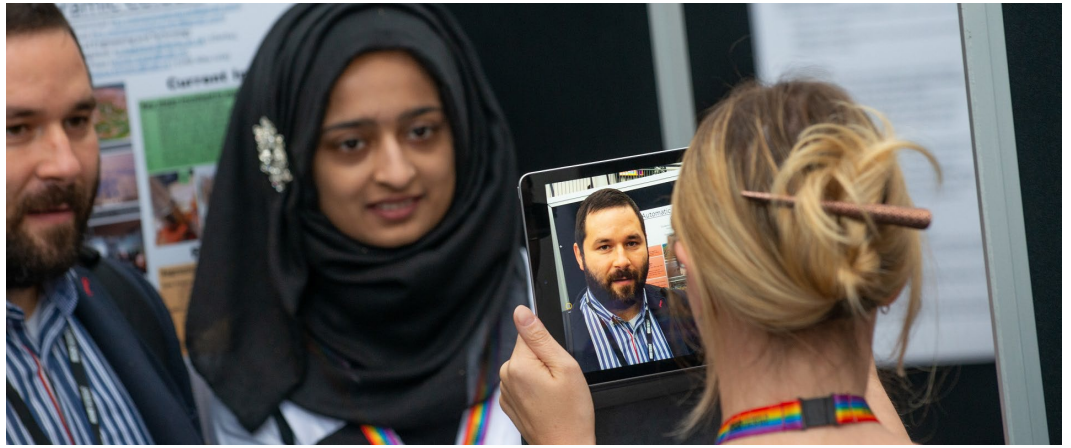
Applied learning: Ceramic color modelling

UNIVERSITY OF DERBY

Develop ceramic glaze recipes which:

- **match** a desired ceramic color
- **correct/fine tune** a ceramic color

Outcome: Taguchi's method, Regression & GUI



Complex recurrences

Definition

The Horadam sequence $\{w_n\}_{n=0}^{\infty}$ is defined by the recurrence

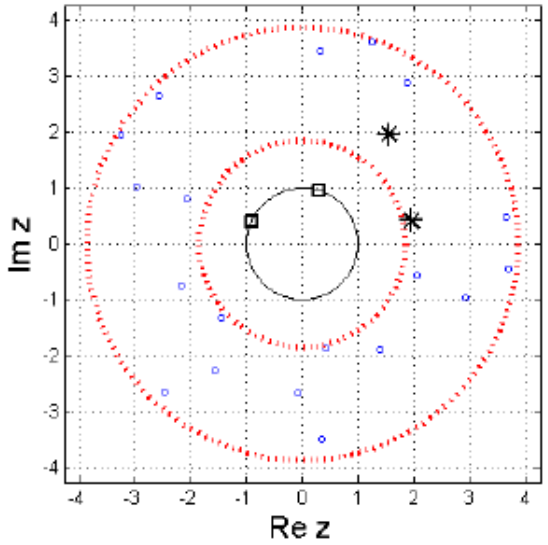
$$w_{n+2} = pw_{n+1} + qw_n, \quad w_0 = a, w_1 = b,$$

where the parameters a, b, p and q are complex numbers.

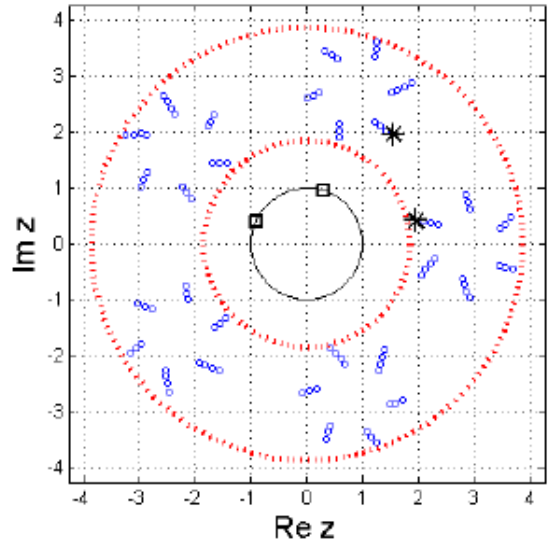
1. Fibonacci sequence: $(a, b) = (0, 1)$ and $(p, q) = (1, 1)$
2. Lucas sequence: $(a, b) = (0, 1)$ and $(p, q) = (1, -1)$
3. Characteristic polynomial: $P(x) = x^2 - px - q$
4. **generators**: the roots z_1, z_2 of $P(x)$
5. Terms can be visualised in the complex plane!

Horadam sequence iterations

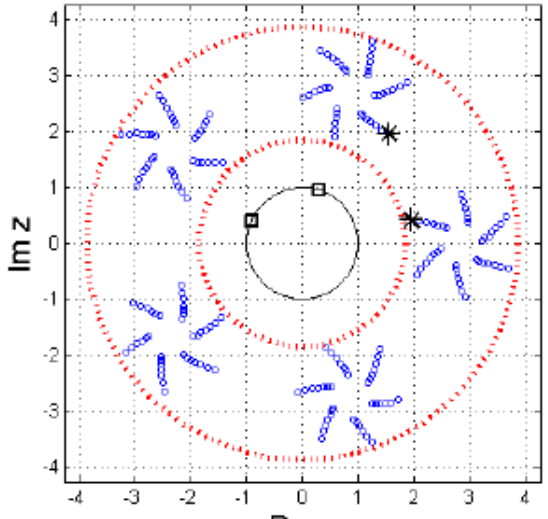
20 terms



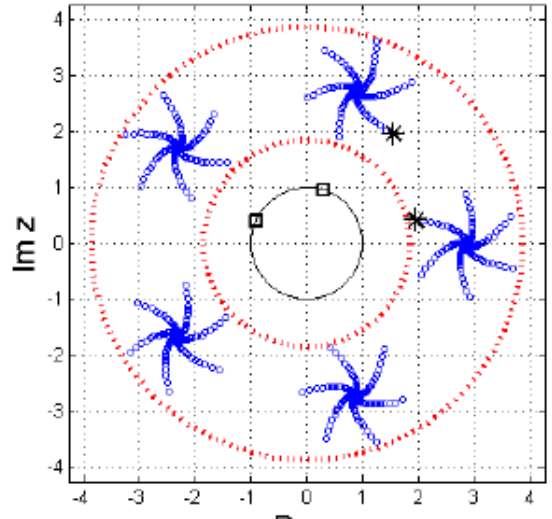
100 terms



200 terms



1000 terms



E. Dijkstra: "A picture may be worth a thousand words, a formula is worth a thousand pictures".

A mini-atlas of non-periodic Horadam patterns

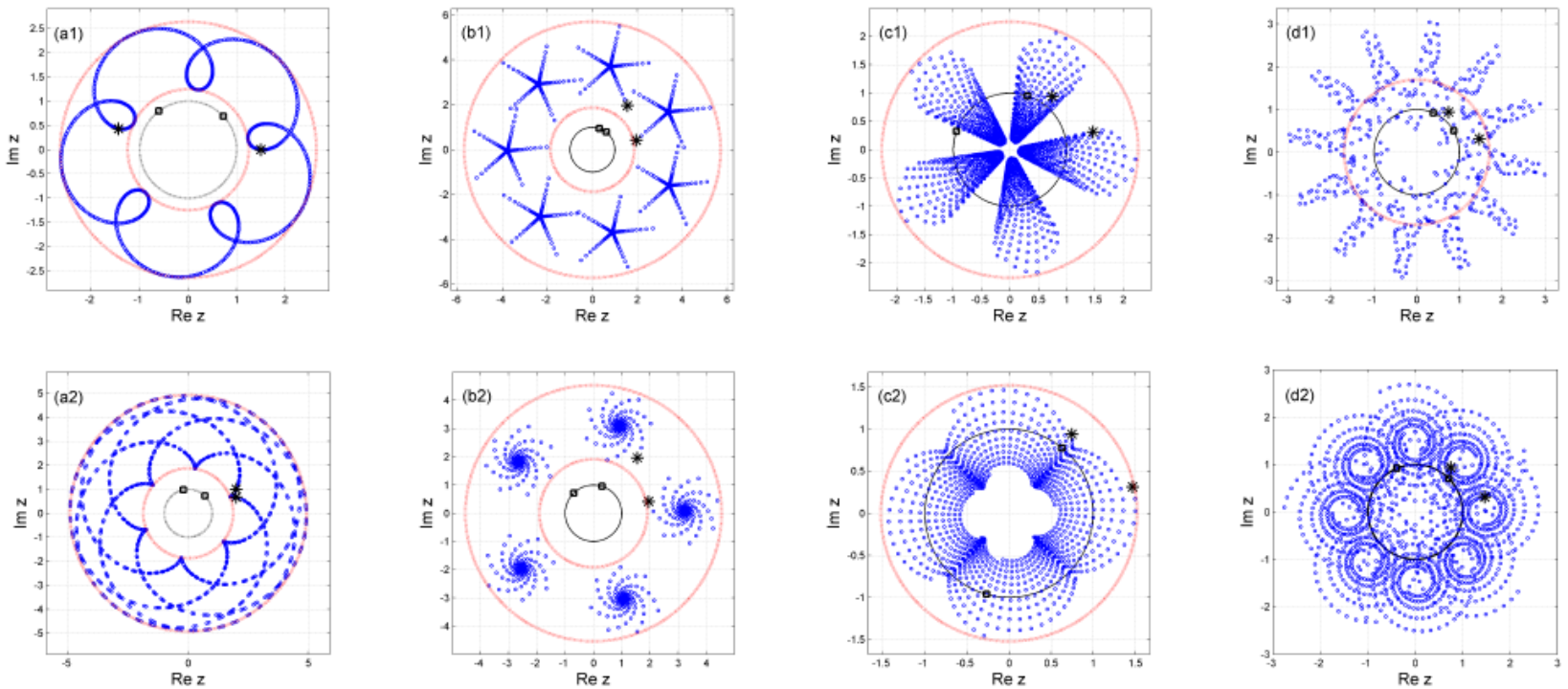
For distinct $z_1 = r_1 e^{2\pi i x_1}, z_2 = r_2 e^{2\pi i x_2}$ $(r_1 \leq r_2)$, the orbit is

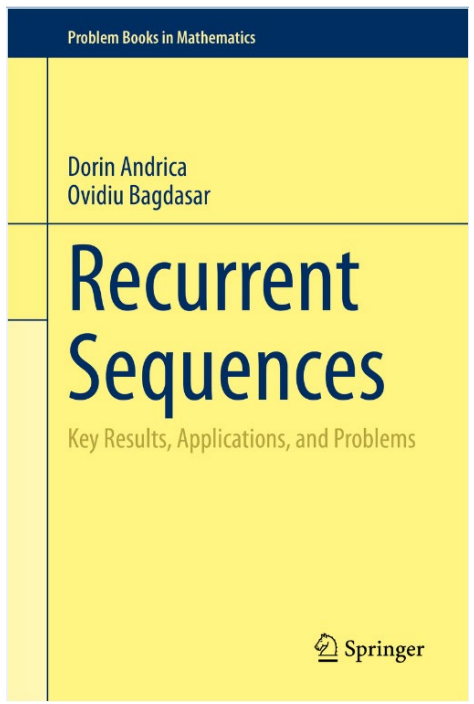
(a) Stable if $r_1 = r_2 = 1$ (unless periodic);

(b) Quasi-convergent if $0 \leq r_1 < r_2 = 1$;

(c) Convergent if $0 \leq r_1 \leq r_2 < 1$;

(d) Divergent if $r_2 > 1$.





Kasner triangles

Problem statement. Let α be real number and $A_0B_0C_0$ a triangle.

Construct the triangle $A_1B_1C_1$ such that A_1, B_1 and C_1 divide the segments $[B_0C_0]$, $[C_0A_0]$ and $[A_0B_0]$, respectively, in the ratio $1-\alpha : \alpha$. Continuing this process we obtain a sequence of triangles $A_nB_nC_n$, $n \geq 0$.

These terms are called Kasner triangles (after E. Kasner (1878-1955)).

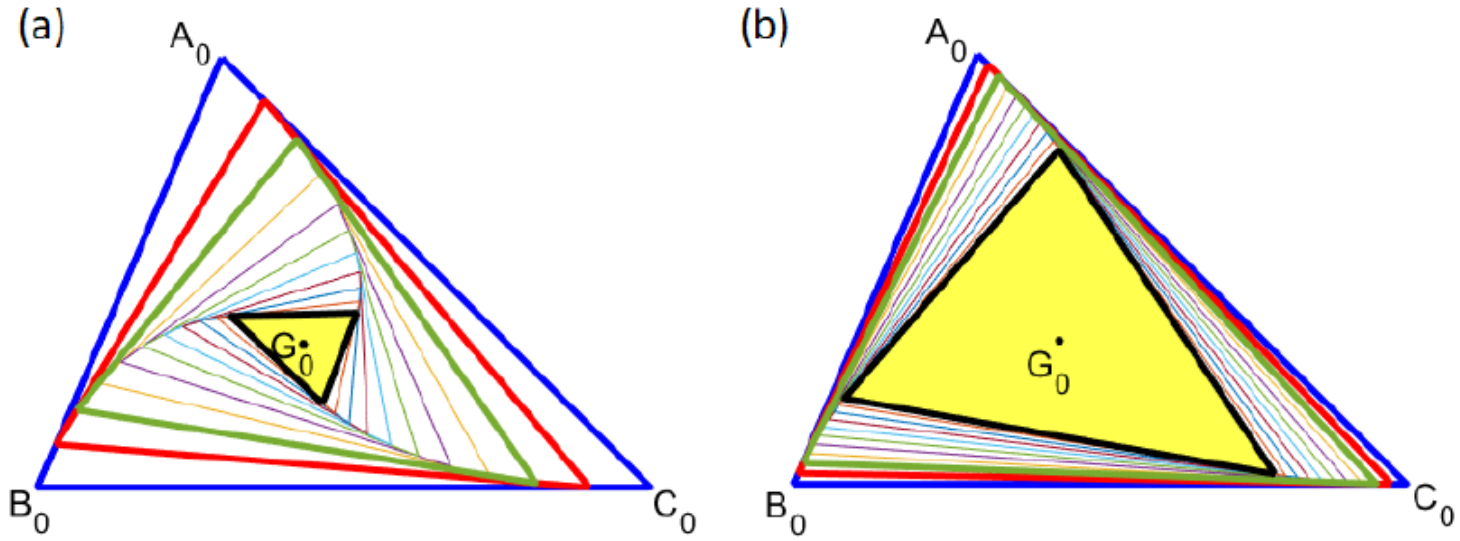


Figure: Sequence of Kasner triangles $A_nB_nC_n$ with $n = 0, \ldots, 10$, computed for (a) $\alpha = 0.1$; (b) $\alpha = 0.025$.

Andrew Wiles:

"The definition *of a good mathematical problem is the mathematics it generates rather than the problem itself*"

Kasner triangles with complex parameter - Formulae

The system (1) can be written in matrix form as

$$X_{n+1} = \begin{pmatrix} a_{n+1} \\ b_{n+1} \\ c_{n+1} \end{pmatrix} = \begin{pmatrix} 0 & \alpha & 1-\alpha \\ 1-\alpha & 0 & \alpha \\ \alpha & 1-\alpha & 0 \end{pmatrix} \begin{pmatrix} a_n \\ b_n \\ c_n \end{pmatrix} = TX_n,$$

where $X_n = (a_n, b_n, c_n)^T$, $n \geq 0$. In this notation one can write

$$X_n = T^n X_0.$$

The characteristic polynomial of T is

$$p_T(u) = (u-1)\left(u^2 + u + 3\alpha^2 - 3\alpha + 1\right),$$

whose roots are $u_0 = 1$ and for $\omega = \exp\left(\frac{2\pi i}{3}\right)$ we have

$$u_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i - \alpha\sqrt{3}i = \omega - \alpha\sqrt{3}i,$$

$$u_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i + \alpha\sqrt{3}i = \omega^2 + \alpha\sqrt{3}i.$$

Dynamical properties: Convergent orbits (1)

Theorem

1° *The sequence of triangles* $(A_nB_nC_n)_{n\geq 0}$ *is convergent if and only if* $\alpha \in D_1 \cap D_2$.

2° *When the sequence* $(A_nB_nC_n)_{n\geq 0}$ *is convergent, its limit is the degenerated triangle at* G_0, *the centroid of the initial triangle* $A_0B_0C_0$.

For $0 < \alpha < 1$ one has $\alpha \in D_1 \cap D_2$, when A_{n+1}, B_{n+1}, C_{n+1} are interior points of $[B_n, C_n]$, $[A_n, C_n]$ and $[A_n, B_n]$, as shown in Figure 5.

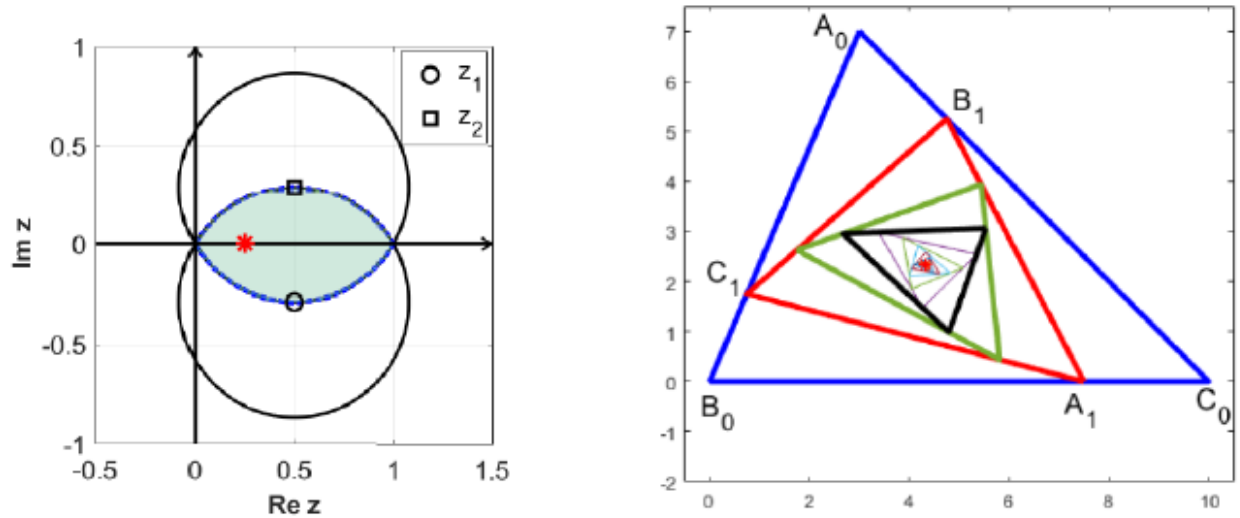


Figure: Convergent orbits (right) obtained for $\alpha = 0.25$ (left).

Dynamical properties: Convergent orbits (2)

Theorem

1° *The sequence of triangles* $(A_nB_nC_n)_{n\geq 0}$ *is convergent if and only if* $\alpha \in D_1 \cap D_2$.

2° *When the sequence* $(A_nB_nC_n)_{n\geq 0}$ *is convergent, its limit is the degenerated triangle at* G_0, *the centroid of the initial triangle* $A_0B_0C_0$.

On the other hand, when the parameter $\alpha \in D_1 \cap D_2$ is not real, the orbit is convergent, but the points are not aligned any more, as in Figure 6.

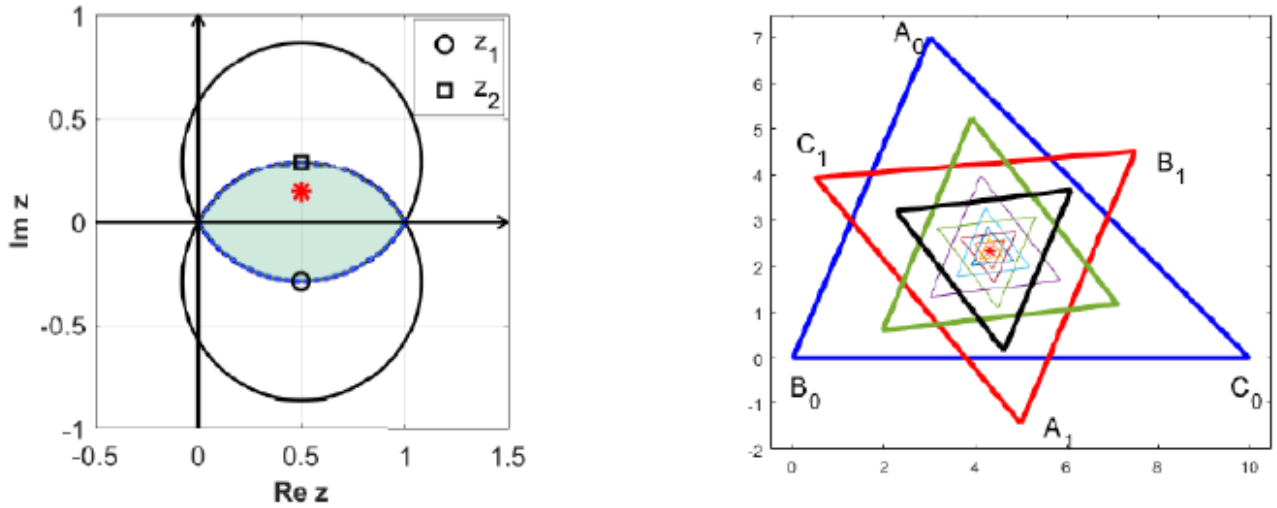


Figure: Convergent orbits (right) obtained for $\alpha = \frac{1}{2} + \frac{\sqrt{3}}{12}i$ (left).

If people do not believe that mathematics is simple, it is only because they do not realize how complicated life is.

(John von Neumann)

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